STRATIFIED FLOW PAST A HILL: APPLICATION OF THE DIVIDING STREAMLINE CONCEPT

By

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Dividing Streamline Concept (DSLC)

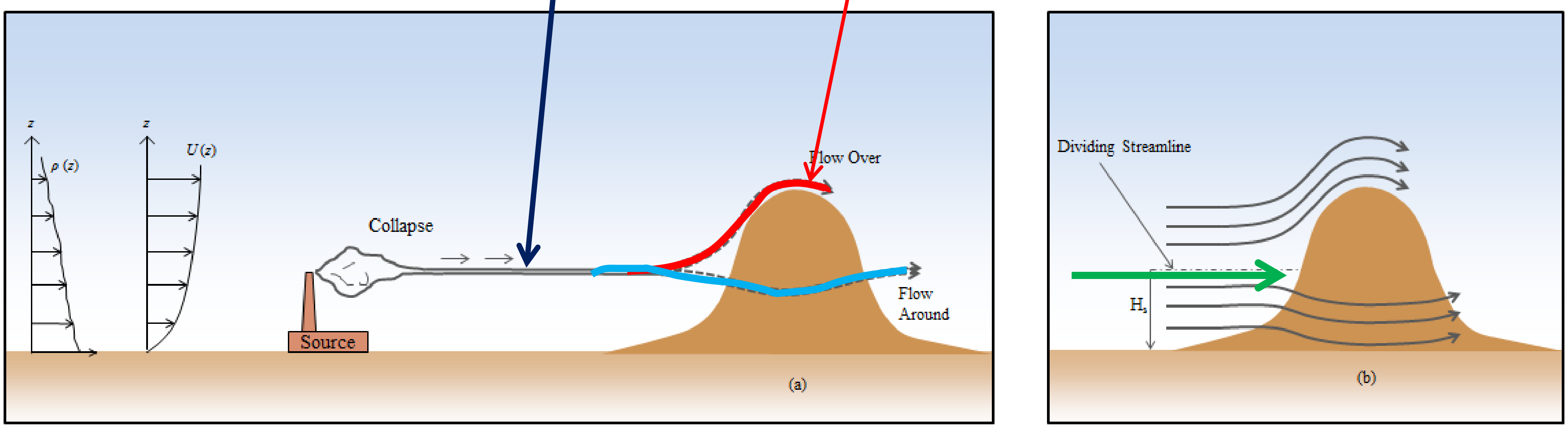
Based on energy arguments:

Sheppard (1956)

“Under what conditions will an airstream rise over a mountain range?”

Sheppard’s Equation:

$$\underbrace{\frac{1}{2}\rho\left(U_0(H_s)\right)^2}_{\text{The kinetic energy of the parcel far upstream at elevation } H_s} = \underbrace{g\int_{H_s}^{h}(h-z)\left(-\frac{\partial\rho}{\partial z}\right)dz}_{\text{The potential energy gained by the parcel in being lifted from the dividing streamline H, to the top of the hill h through the density gradient } \partial\rho/\partial z} \qquad (1)$$



A conceptualization of source pollution within a stably stratified flow collapsing into a thin layer, and becoming entrained in the flow.

Dividing Streamline Concept (DSLC)

$$\frac{1}{2}\rho\big(U_0(H_s)\big)^2 = g\int_{H_s}^{h}(h-z)\left(-\frac{\partial\rho}{\partial z}\right)dz$$

This generally requires *iterative* solving…

Sheppard's formula can be simplified assuming:

1. Constant density gradient β
2. Uniform approach velocity profile, U_∞

$$\frac{1}{2}\rho(U_\infty)^2 = g\int_{H_s}^{h}(h-z)(-\beta)dz$$

Sheppard's Formula

$$\boxed{\frac{H_s}{h} = 1 - \gamma\, Fr}$$

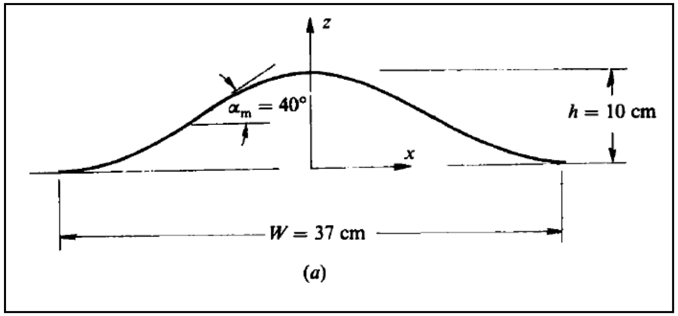
$$Fr = \frac{U_\infty}{N_\infty h} < 1$$

Assuming total energy transfer, $\gamma = 1$

Lends itself to laboratory experiments, *ex.* Stratified towing tank

DSLC: Experiments

Sinusoidal Mountain Study



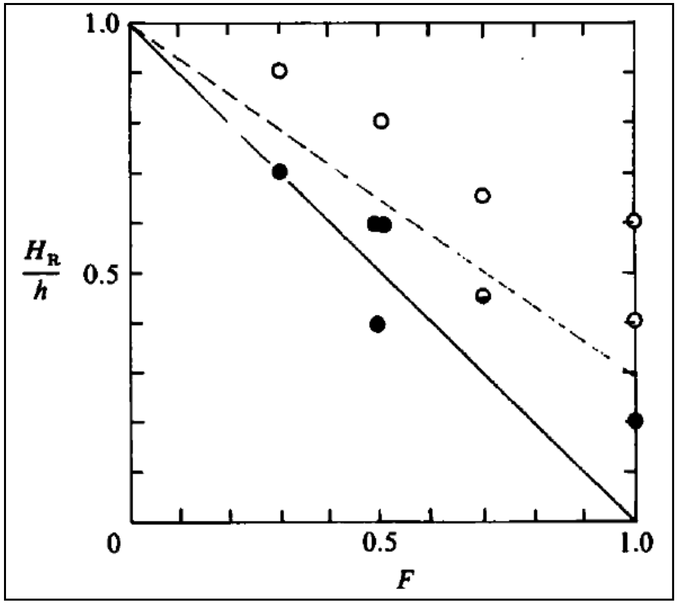
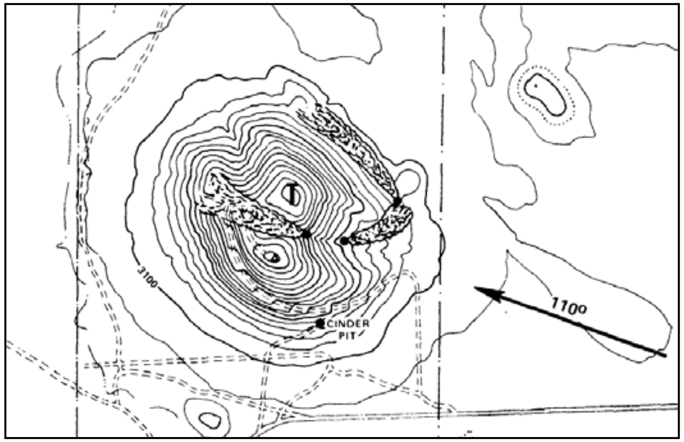
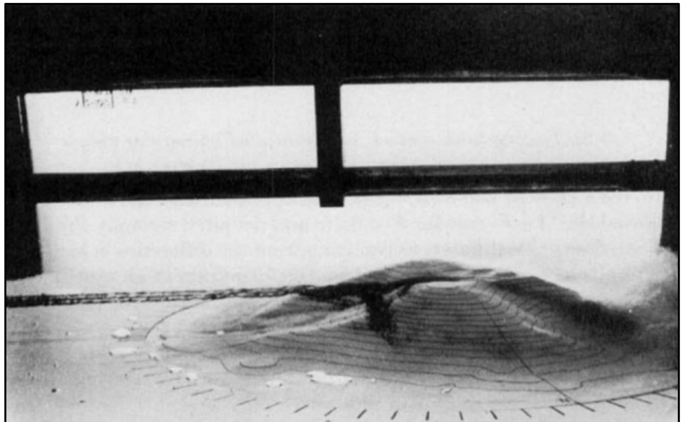


Figure : Simple sinusoidal 3-D ridge in a stratified tow tank, adopted from Hunt et al. (1980); Snyder et al., (1985).

Cinder Cone Butte, ID



Adopted from Snyder et al. (1980)



Adopted from Snyder et al. (1985)

Figure : Field and laboratory experiments conducted on Cinder Cone Butte, ID; focused on dividing streamline concept.

Theoretical extensions: Log Vel. Profile

Remember assumptions:

1. Constant density gradient β
2. Velocity profile: $\boxed{u(z) = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right)}$

)

Theoretical extensions: Log Vel. Prof

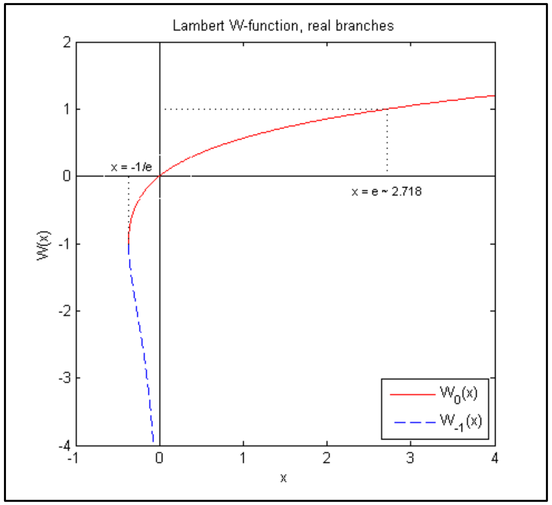
Taking the exponential of each side: $$\frac{H_s}{z_0} e^{\frac{N\kappa}{u_*}H_s} = e^{\frac{N\kappa}{u_*}h}$$

$$\left(\frac{N\kappa}{u_*}H_s\right) e^{\left(\frac{N\kappa}{u_*}H_s\right)} = e^{\left(\frac{N\kappa}{u_*}h\right)}\left(\frac{N\kappa}{u_*}z_0\right)$$

$$x = y e^{y}$$

The Lambert-W function

$$\boxed{y = W(x)}$$



$$H_s \frac{N\kappa}{u_*} = W\left(e^{\left(\frac{N\kappa}{u_*}h\right)}\left(\frac{N\kappa}{u_*}z_0\right)\right)$$

$$\boxed{H_s = \frac{u_*}{N\kappa} W\left(e^{\left(\frac{N\kappa}{u_*}h\right)}\left(\frac{N\kappa}{u_*}z_0\right)\right)}$$

Theoretical extensions: Log Vel. Prof

buoyancy lengthscale $$L_b = \frac{u_*}{N\kappa}$$

$$\boxed{\frac{H_s}{L_b} = W\left(\boxed{\frac{h}{L_b}}\boxed{\frac{z_0}{h}}e^{\boxed{\frac{h}{L_b}}}\right)}$$

$$\frac{H_s}{h} = 1 - \gamma\ Fr\ (Sheppards\ Formula)$$

Experimental Site

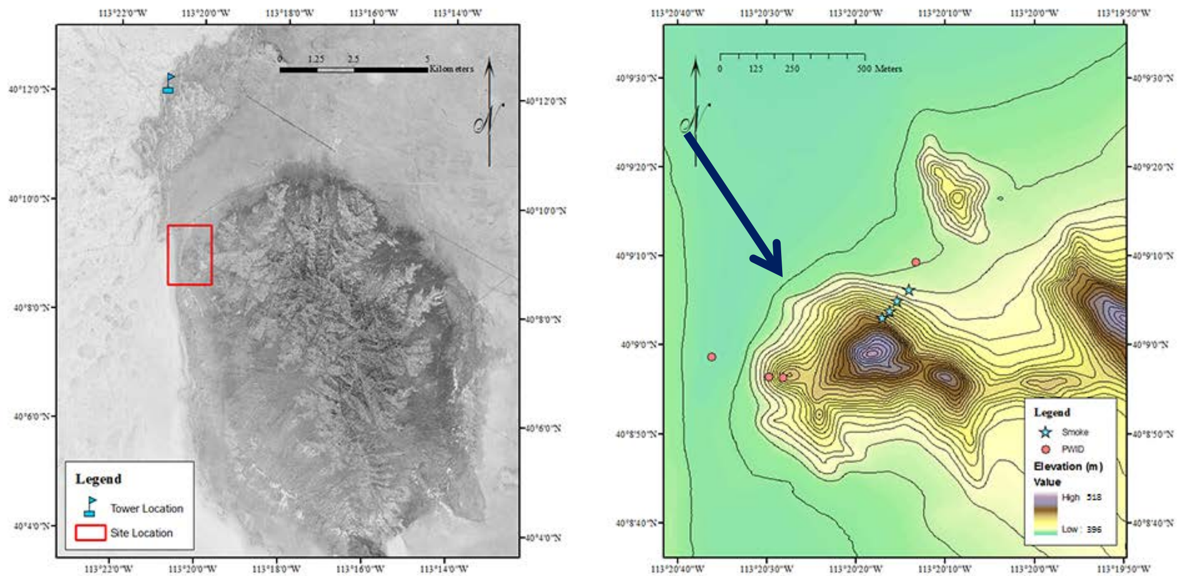


Figure 17: (*Left*) High resolution 10*m* orthoimagery of Granite Mountain, portraying the location of the instrumentation tower and smoke visualization site. (*Right*) Ten times magnification of the smoke visualization site; the contours are presented at 5*m* intervals (DEM data obtained from USGS, 2013).

32m NW tower:

- 5 81000 R.M. Young ultrasonic sonic anemometers (20*Hz*) and Campbell Scientific HMP 45 Temperature probes (1*Hz*)

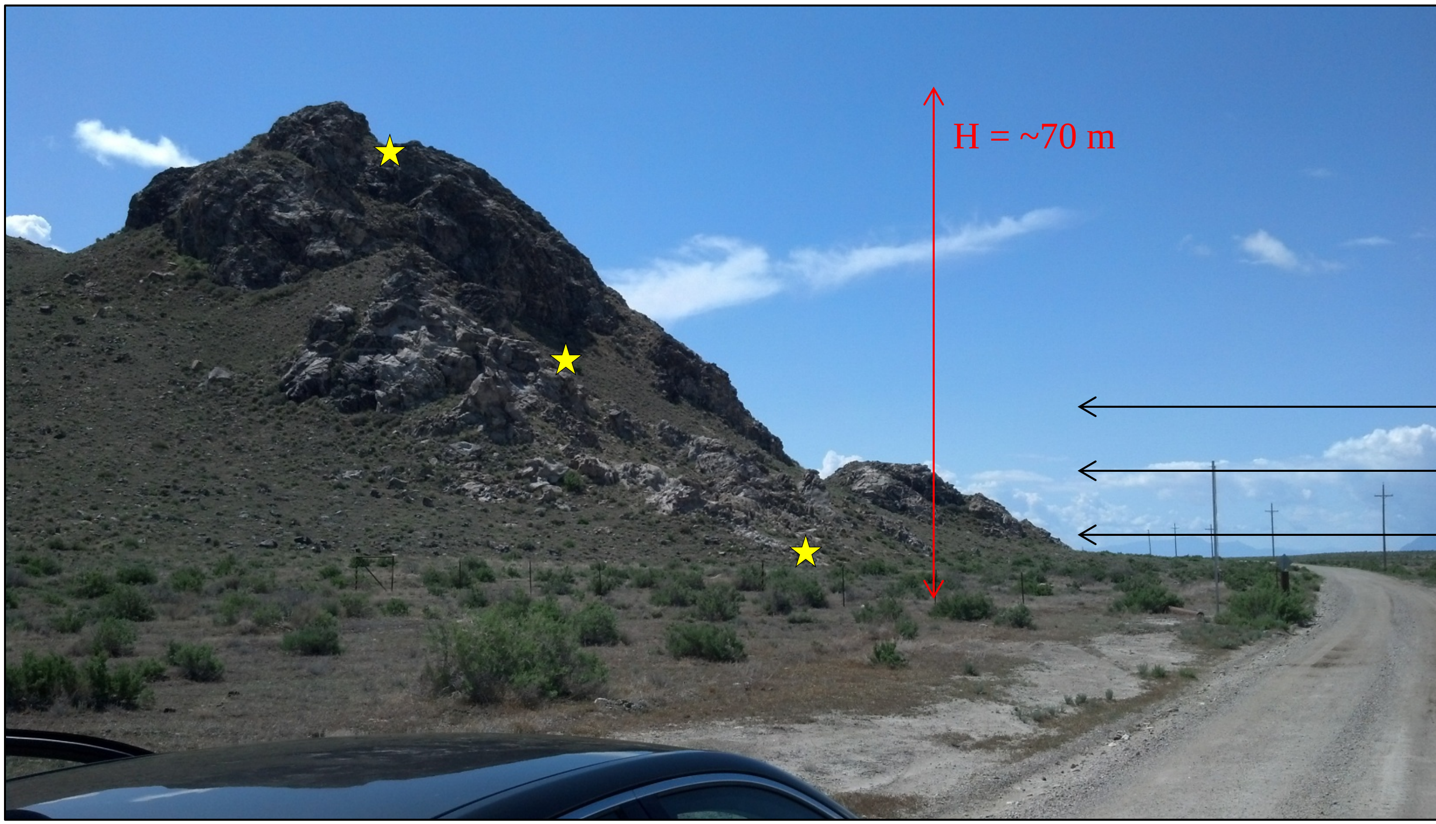
PWIDs:

- 05103 R.M. Young mechanical wind sensors
- HMP45 probes

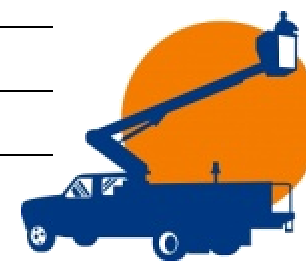
Hill of Interest

Note that camera view is North-Northwest.

★ = The location of a simultaneous smoke canister release.

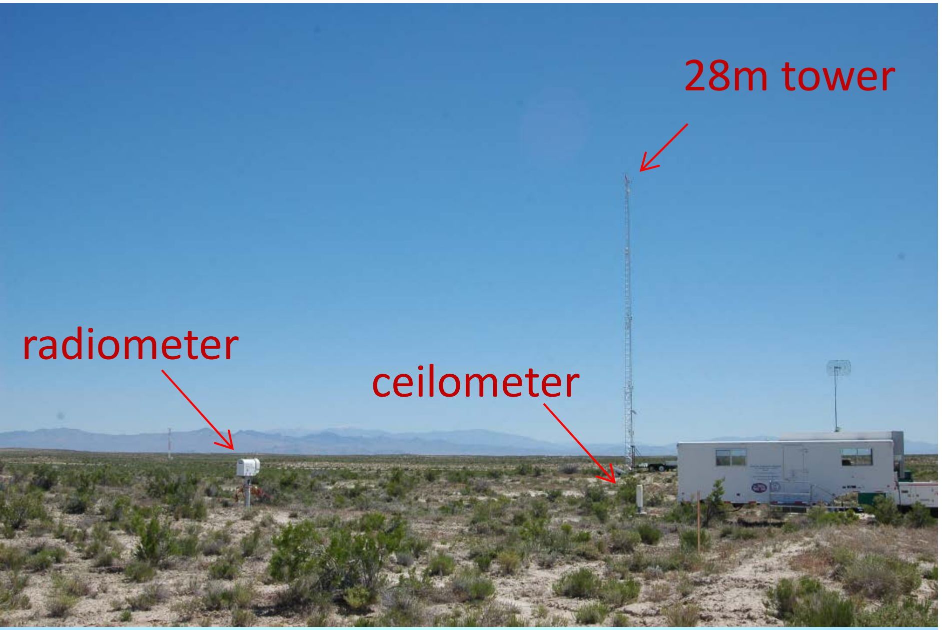


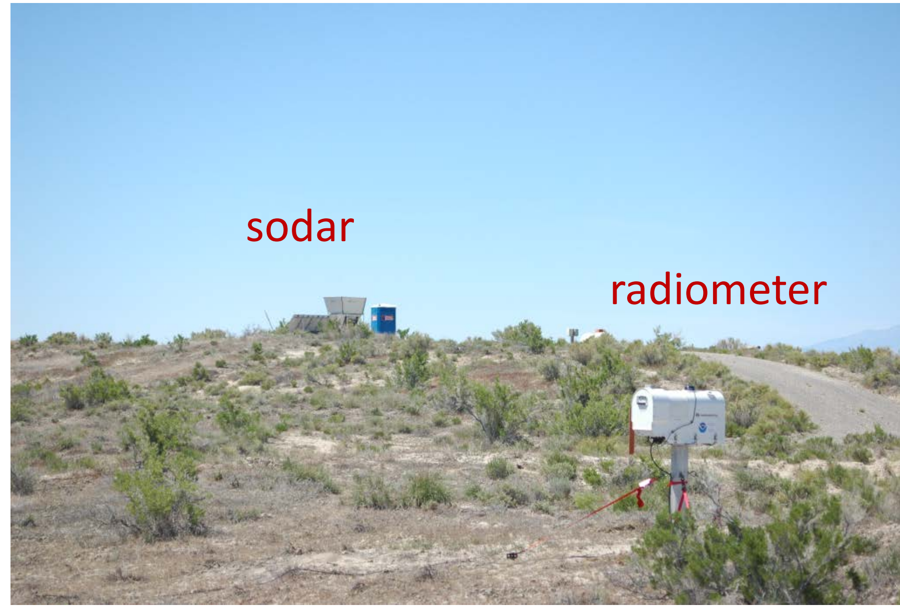
Smoke streamlines at varying elevations.

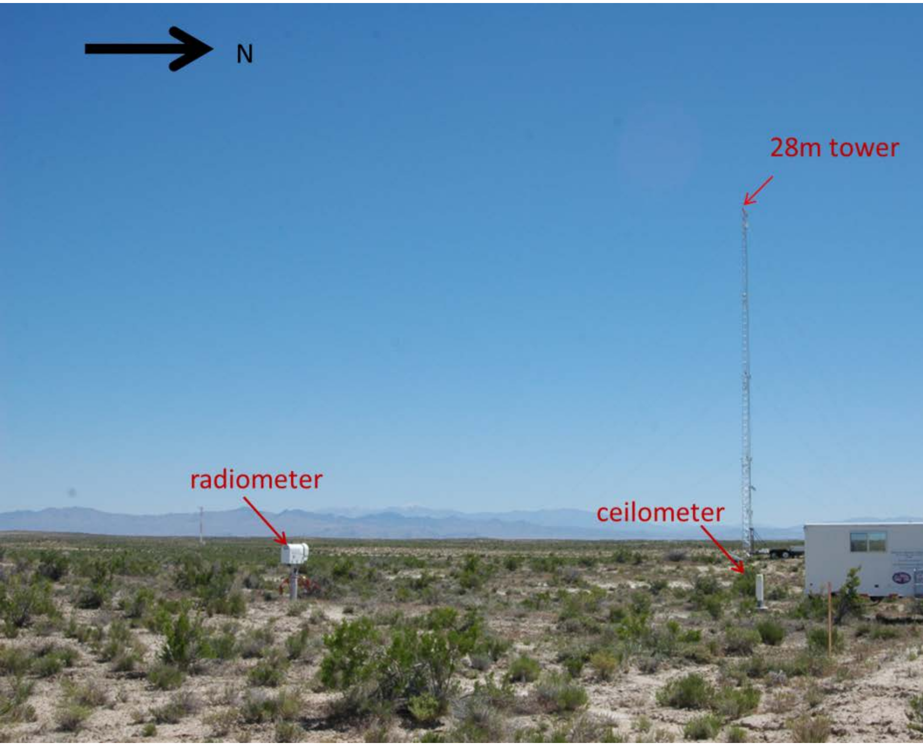


Bucket truck with smoke generator.

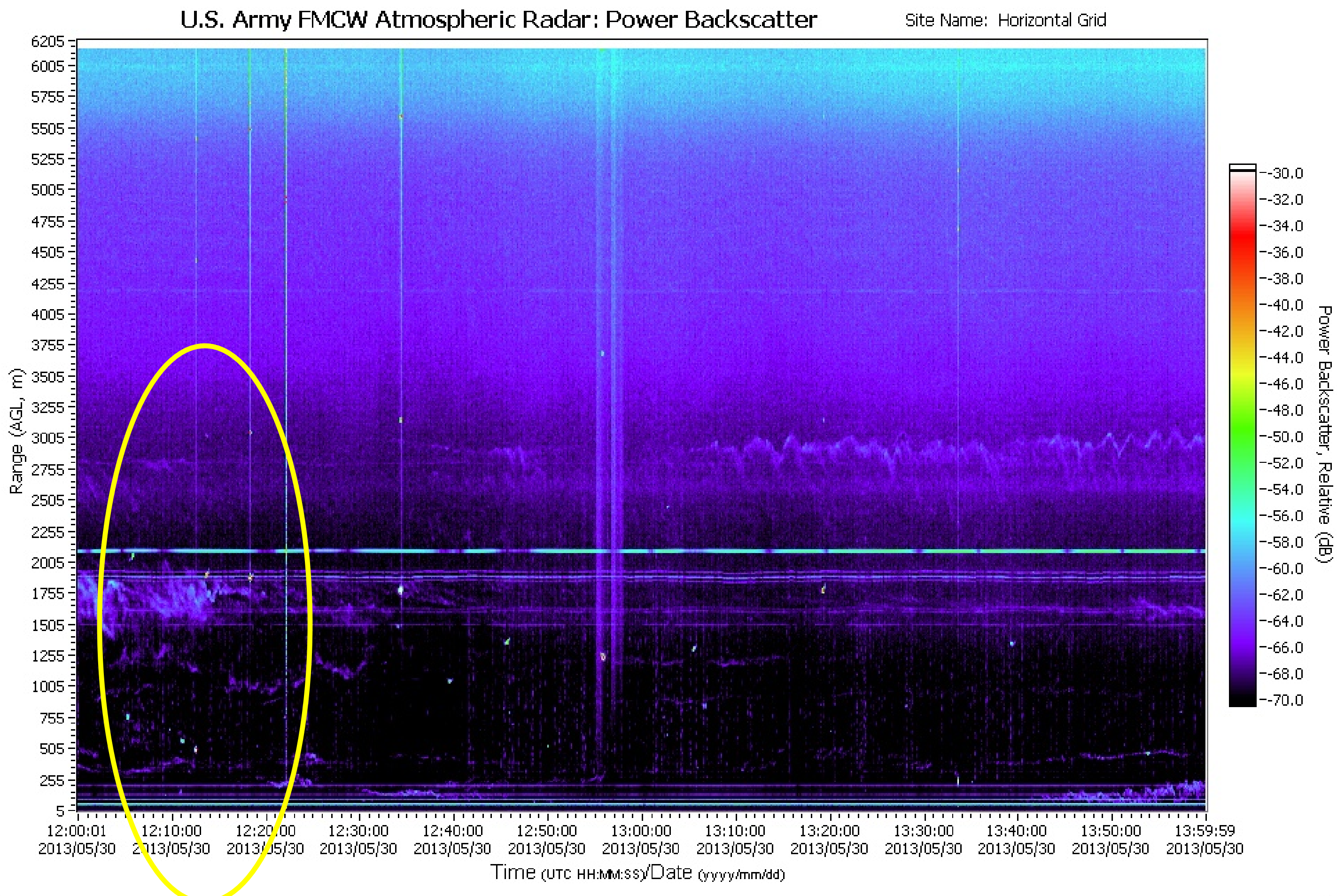
NW site Instrumentations



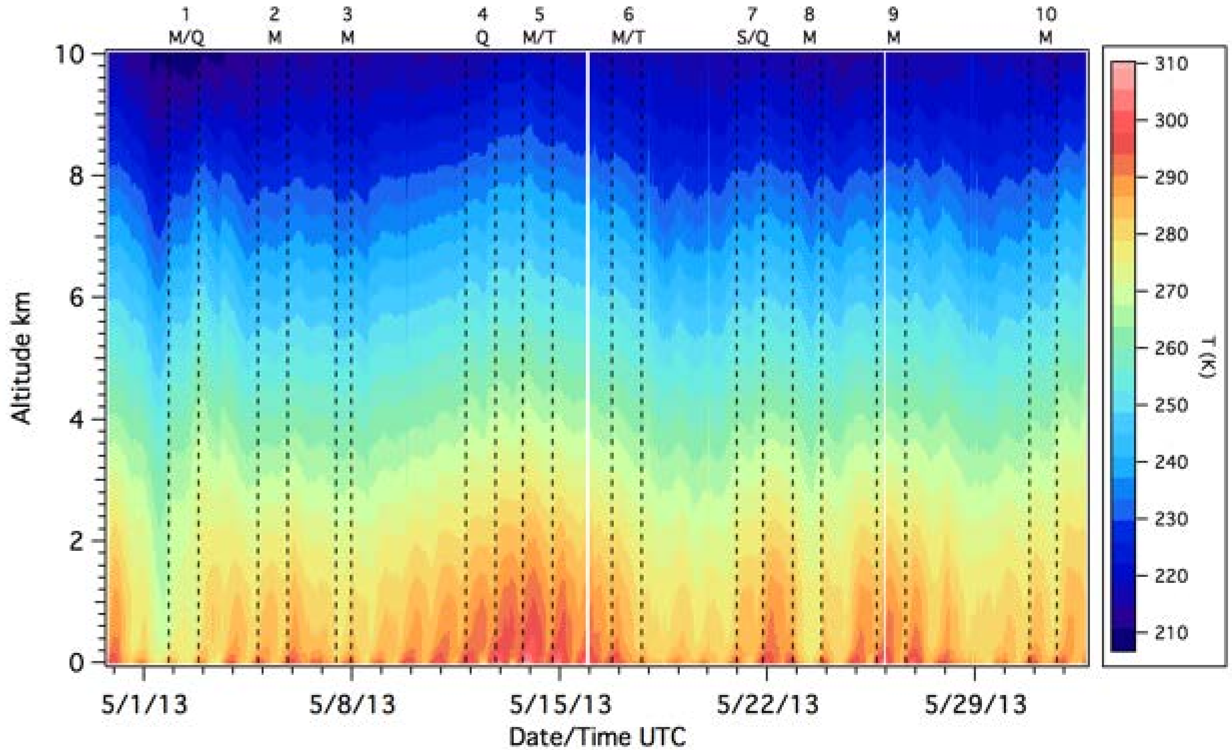




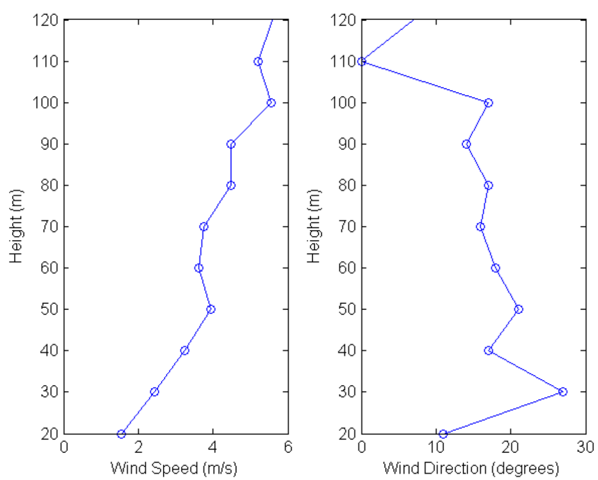
FM-CW Radar



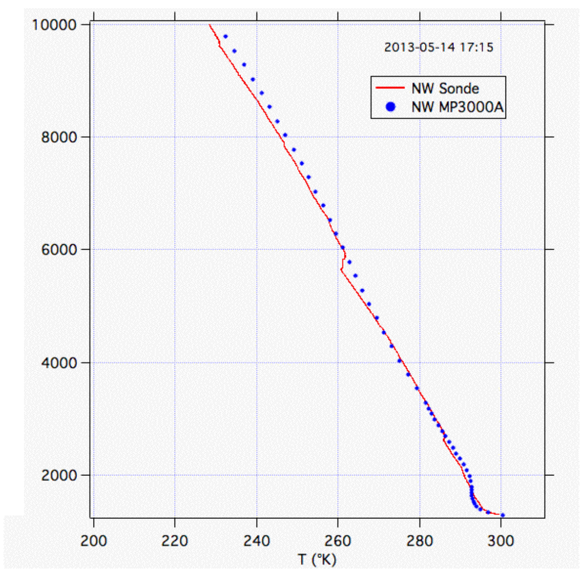
Latitude:	Longitude:	Last Tx Pwr (Watts)	Resolution (meters):	Sweeps per Scan:	Receiver Gain (dB):	Range Corr. Filter (kHz):	TS Window:	Averaging:	GCF On?:	GCF Coef:	TS Median:	Spectra Median:	Filename (yymmddHHMM.jpg)
40 11.80 N	113 10.06 W	125.9	6.0	64	30	100	Hanning	RMS averaging	Yes	0.4	0	0	1305301200-1359.jpg



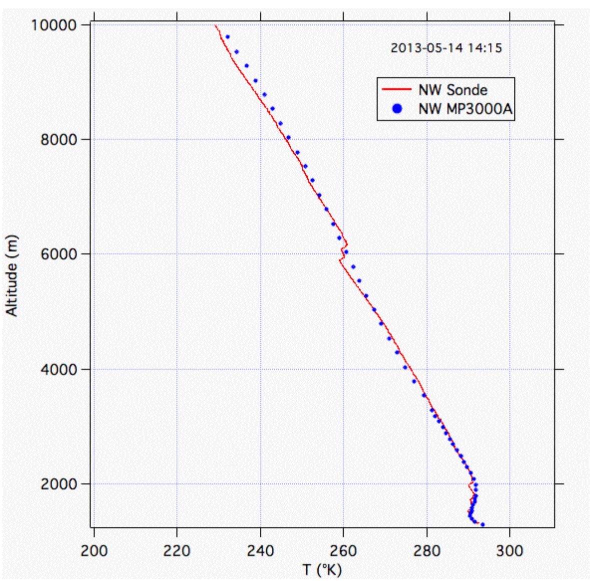
Radiometer profiles



Sodar Profiles



chart



Radiometer evaluation against radiosondes

Attempted Smoke Releases

Table 1: Smoke release trials, resulting in a control case (purple), and stratified case (green).

Date	Start Time	Smoke Method	Quantity	Location	Weather Conditions	
					Cover	Onsite WD
5/20/2013	6:30 PM	Smoke Generator	3 gallons mineral oil	NW Slope	Clear	*N/NE*
5/22/2013	1:54 PM	Smoke pot (white)	1 pots	NW Slope	Overcast	*N*
5/23/2013	6:04 AM	Red Smoke Grenade	3 grenades	NW Slope	Clear	*N/NE*
5/23/2013	7:00 AM	Smoke pot (white), Crane	3 pots	NW Slope	Clear	*N/NE*
5/23/2013	8:00 PM	Smoke Generator	2 gallons mineral oil	N Slope	Clear	*N/NE*
5/24/2013	7:00 AM	Ag Spray	A few minutes running	NE Slope	Clear	*S/SW*
5/28/2013	8:00 AM	AG Spray	15 minutes running	NE Slope	Partly Cloudy	*N/NE*
5/28/2013	8:08 AM	Red Smoke Grenade	2 grenades	NE Slope	Partly Cloudy	*N/NE*
5/30/2013	6:00 AM	AG Spray	A few minutes running	NE Slope	Clear	*N/NE*
5/30/2013	6:15 AM	Red Smoke Grenade	4 grenades	NE Slope	Clear	*N/NE*
5/30/2013	6:30 AM	Smoke pot (white), Crane	3 pots	NE Slope	Clear	*N/NE*

Smoke Release Analysis #1

May 30th, 2013 – *Stratified Flow*

Red **Smoke Canisters** (06:15 MDT):

- 4 simultaneous ground releases at approximately $0h, 0.08h, 0.42h$ and $0.92h$
- Neutrally buoyant, burning duration of approximately two minutes
- Saddle point = hill top (Snyder et al., 1980)

White **Smoke Canisters** (06:20 MDT):

- 3 simultaneous ground releases at approximately $0.08h, 0.42h$ and $0.92h$
- 1 elevated release $0.37h$ above the ground, upstream
- Neutrally buoyant, burning duration of approximately five minutes
- Saddle point = hill top (Snyder et al., 1980)

May 30th *(Stratified):* Measured Profiles

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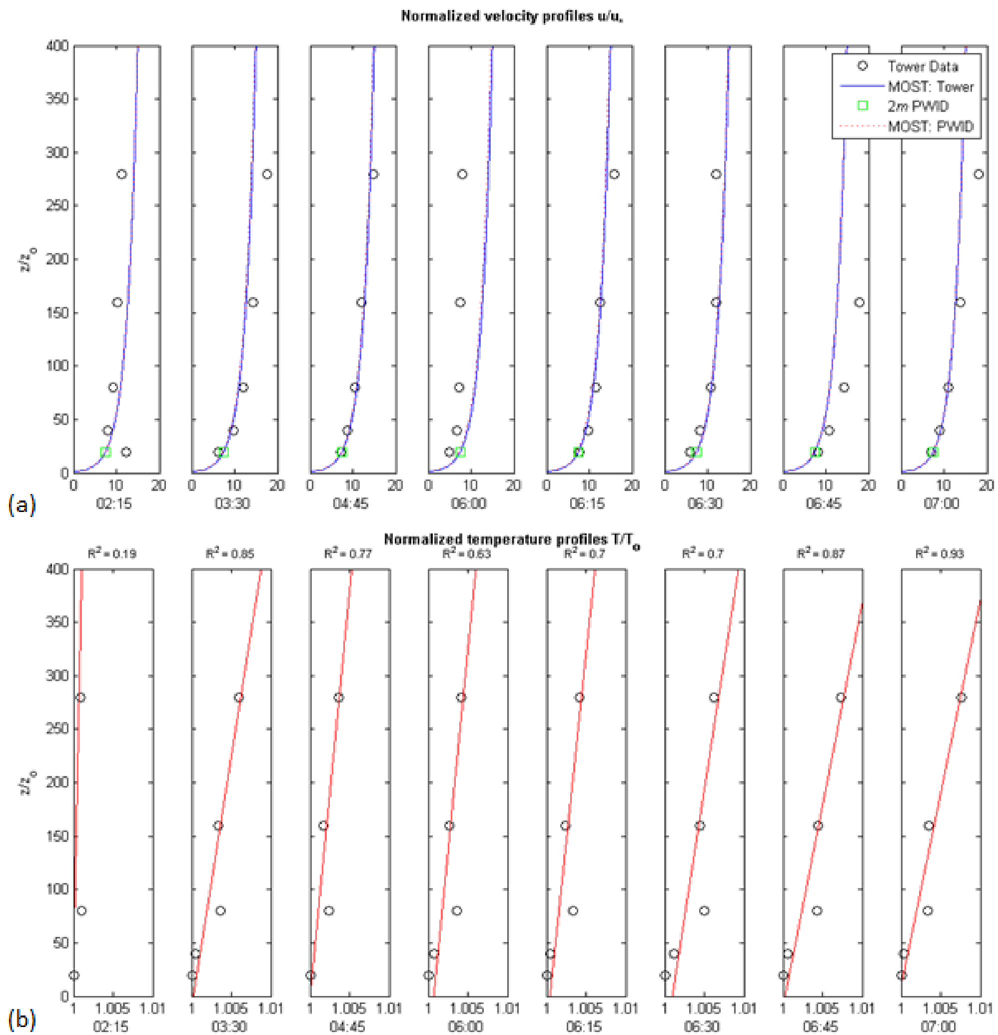


Figure (*a*) Normalized velocity profiles calculated from the 32m tower, and the 2m PWID. (*b*) The temperature profiles measured by the tower HMP 45 probes with linear fits, and coefficients of correlation. Note that times are presented in MDT, and that flow visualization started at 6:15AM MDT.

$$\frac{H_s}{L_b} = W\left(\frac{h}{L_b}\frac{z_0}{h}e^{\frac{h}{L_b}}\right)$$

May 30th *(Stratified):* Visualization

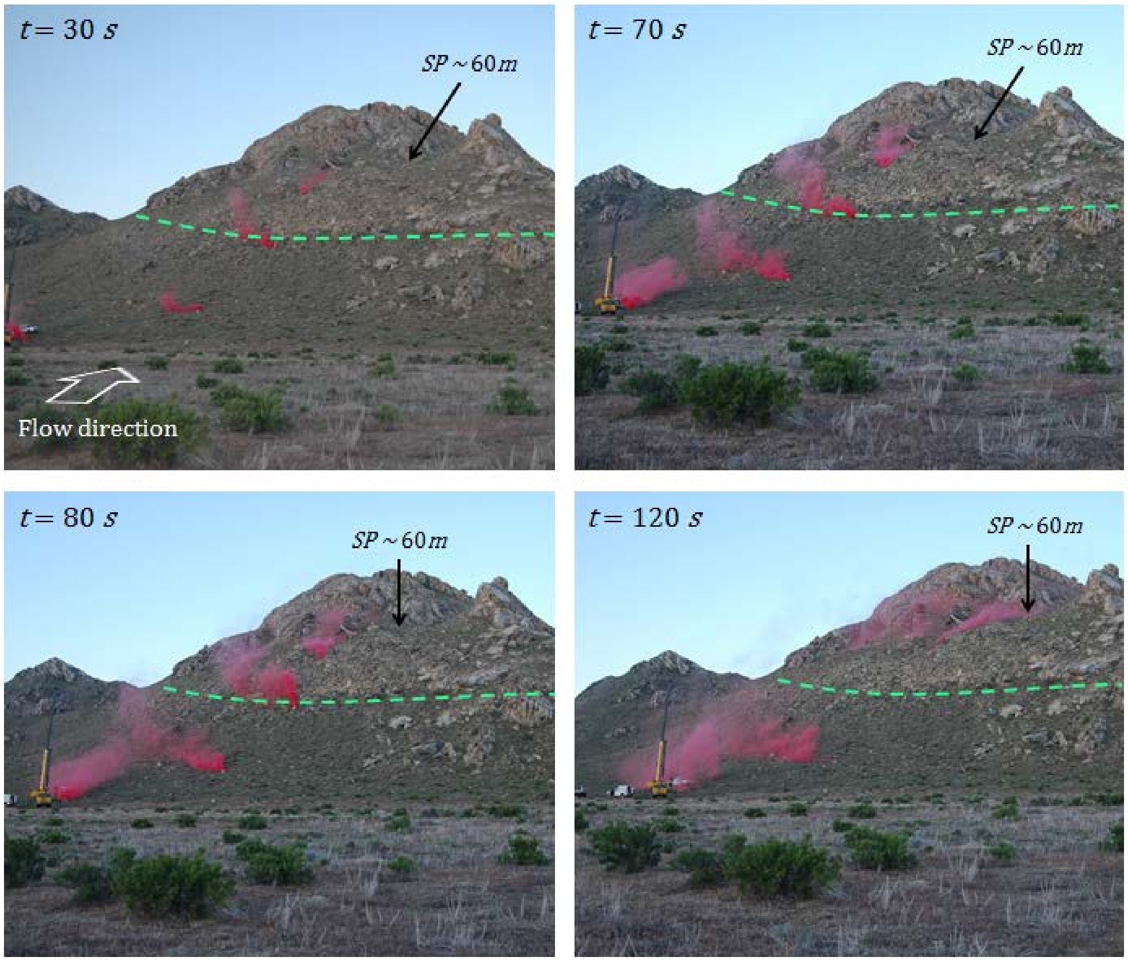


Figure 25: Red smoke release during May 30th, 2013. The still photos are taken at approximately 30, 70, 80, and 120 seconds after the release of the smoke canisters. The dashed green line is a visual guide to approximate .

Movie 1: Red smoke release.

May 30th *(Stratified):* Visualization

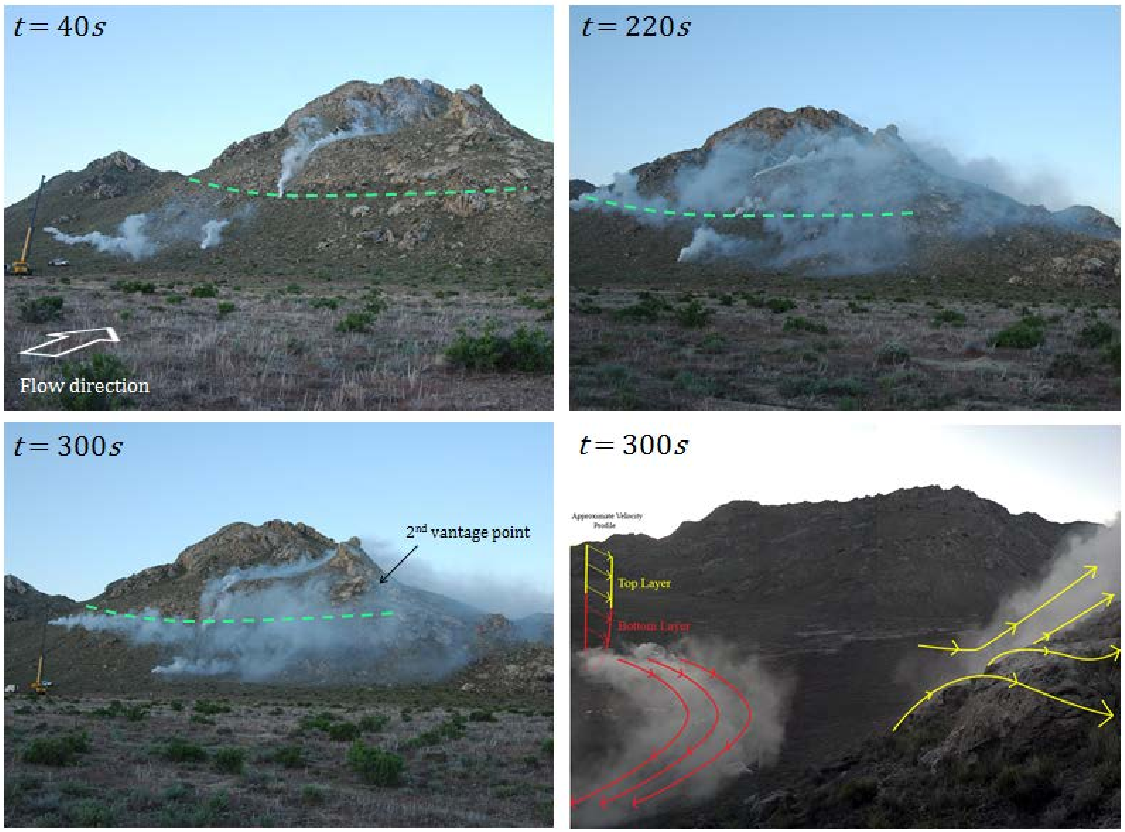


Figure 26: White smoke release during May 30, 2013. The still photos are taken at approximately 40, 220, and 300 seconds after the release of the smoke canisters. The vantage point of the last photograph is portrayed in the photograph before it. The streamlines and velocity profile in the final photograph is for illustration only.

Movie 2: White smoke release.

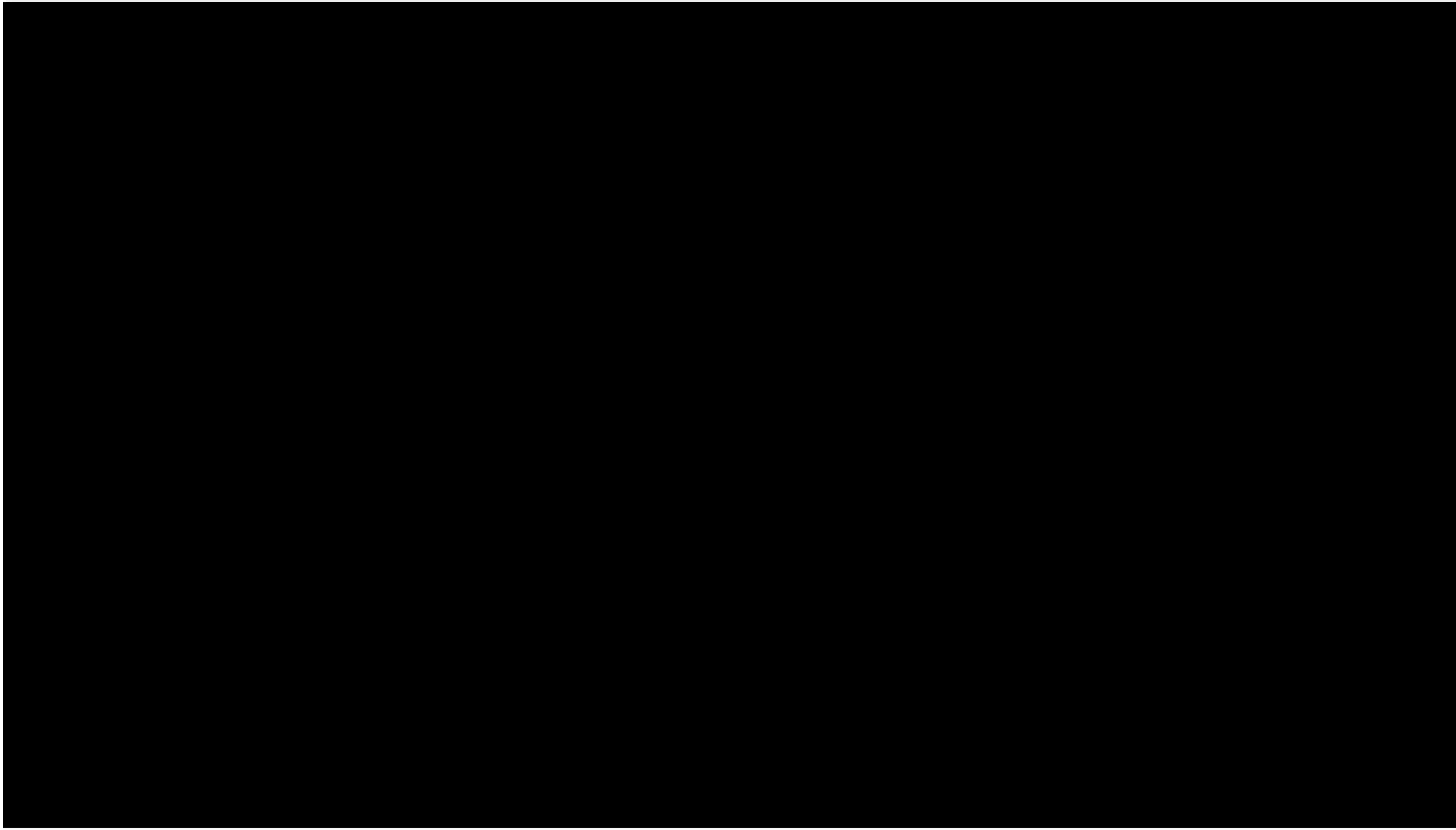
Conclusions

1. Application of log velocity profile to Sheppard's formula → explicit solution for H_s, which utilizes the Lambert-W function

2. This new representation is based on a buoyancy length scale, L_b (previously used *Fr* scaling)

3. Field observations support the derived expression

May 30th *(Stratified):* Movie



Movie 3 : Full flow visualization movie.

Thank you